The Mathematical Association

Primary Mathematics Challenge

Bonus Paper

Answers and Notes February 2012

These notes provide a brief look at how the problems can be solved. There are sometimes many ways of approaching problems - not all can be given here. Suggestions for further work based on some of these problems are also provided.

P1	A	impossible	3 or 4 eggs in a minute, but surely not 1000! That must be impossible.
P2	B	3	The total of the ages now is 88, that is 12 years under 100. The total increases by 4 each year, so their ages will add up to 100 in $12 \div 4 = 3$ years.
1	**C**	8	$2012 = 2 \times 2 \times 503$. Factors are 2, 4, 503 and 1006, but not 8.
2	**C**	3	The horses need 27 gallons a day. That means Sue needs 7 containers. But she only has 3, so she has to make 3 trips in the car – the last trip with only one container.
3	**D**	154	Two numbers adding to 25 with a difference of 3 must be 11 and 14. The product of their ages is $11 \times 14 = 154$.
4	**E**	64	The square numbers are 4, 9 and 64. The cube numbers are 8, 27 and 64. The only one which is both is 64.
5	**B**	$p - q + r$	The algebraic lengths might worry some pupils, but replacing them with numbers might make it clear that the distance x is $p - q + r$.
6	**C**	April	If 13th January is a Friday, the 13th February will be a Monday (January has 31 days which is four weeks and three days). February 2012 is a leap year (29 days), so 13th March will be a Tuesday. March has 31 days, so 13th April will move 3 days further and will be a Friday. This is the 'next' Friday the thirteenth.
7	**D**	100 000	6650km ÷ 61m is approximately $6300\,000 \div 63 = 100\,000$.
8	**C**	7w 3d	Marcus has been absent 20% of 190 days = 38 days. With a school week of 5 days, this is 7 weeks and 3 days.
9	**D**	hexagonal prism	The number of edges for these shapes is A (6), cube (12), square-based pyramid (8), hexagonal prism (18) and octahedron (12).

10	**B**	$3m^2$	Amy's rose bed will be 9 metres long and 1/3 metre wide. Its area will be $9 \times ⅓ = 3m^2$.
11	**D**	4/5 century minus 3 years	John was 13 at the time of the 1948 Olympics, so he was 13 + 52 years old at the millennium, and so will be 13 + 52 + 12 = 77 years old in 2012. That is $4/5^{th}$ of a century minus 3 years.
12	**E**	£360	£10 for 20 injections gives 50p for each. At his improved speed he can give 20 × 36 = 720 injections in 20 minutes. At 50p each he will earn £360.
13	**E**	16	$\sqrt{16} = 4$, so the equation now looks like $4 + \sqrt{n} = 8$. This tells us that $\sqrt{n}$ must equal 4 so n = 16.
14	**D**	a multiple of 30	All prime numbers other than 2 are odd numbers. Multiples of 3 can be odd (3, 9, 15 etc.). Some odd square numbers are 9, 25 and 49. The factors of three are 1 and 3, both odd. But all multiples of 30 are even.
15	**B**	654 ÷ (3 + 21)	A is 654 ÷ 34. B is 654 ÷ 24. C is 654 ÷ 33. D is 654 ÷ 31 and E is 654 ÷ 321. The value of a division sum is largest when the divisor is the smallest, so B is the answer.
16	**E**	630 ml	Aunty needs 15 × 3 × 7 × 2 = 630ml.
17	**C**	Wednesday	Luigi's birthday will be 34 + 43 days after Julian's birthday. That is 77 days which is exactly eleven weeks. So Luigi's birthday will also be on a Wednesday.
18	**B**	20	The smoky bacon sector of the pie chart has an angle of 45º, and is one eighth of the full pie (360º). So the number of people in the survey must be divisible by eight. From the five numbers in the answers, only 20 is not divisible by eight.
19	**C**	18.914	2% of 19.3 is (2 × 19.3) ÷ 100 = 38.6 ÷ 100 = 0.386. Usain Bolt's new time would therefore be 19.3 – 0.386 = 18.914.
20	**A**	5	There are 13 noses and so 13 'creatures'. The number of tails equals the number of dogs so pupils can use trial and error to find the answer. Or we can assume that each nose has 2 legs, giving us 26 legs and leaving 10 legs extra, suggesting 5 'things' which have 4 legs each, i.e. dogs. Otherwise, using algebra, $p + d = 13$ and (for the legs) $2p + 4d = 36$. Subtracting twice the first from the second we get $2d = 10$ so $d = 5$.

21 **E** 4 hours

The small tank holds $3 \times 4 \times 5 = 60$ m^3 of water. If the pipe can fill the tank in 20 minutes, it is giving $60 \div 20 = 3$ m^3 water a minute. The large tank holds $6 \times 12 \times 10 = 720$ m^3 of water. This will take $720 \div 3 = 240$ min = 4 hours.
Alternatively, using the ratio of lengths, breadth and height, the time needed to fill the large tank will be $20 \times 2 \times 3 \times 2 = 240$ min = 4 hours.

22 **D** 40p

All but 10 of the coins are 1p coins, which tells us that the number of 2p and 5p coins adds up to 10. Similarly, the total of the 1p and 5p coins is 10 and the total of the 1p and 2p coins is 10. He therefore has 5 of each coin, giving a total of $(5 \times 1) + (5 \times 2) + (5 \times 5) = 40$p.

23 **E** 11

The second-to-last sentence tells us that the number of cards is a square number; i.e. 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169 or 196 (less than 200). The number we are looking for has to be one more than a multiple of 5, and so has its units digit one or six. It also has to have an odd units digit as it has 'one left over after being arranged in fours'. The number must end in one; i.e. it must be 81 or 121. Of these two, only 121 is one more than a multiple of six. So the number of cards on each side of the square is 11.

24 **D** 31

There are 10 seVens in the unit digit, 10 seVentys, 9 fiVes in the units (as fifteen is not fiveteen) plus eleVen and twelVe.

25 **E** orange

The diagrams show the position of the plates as they are moved according to the rules in the question.

Red	*Green*	*Orange*	*Blue*	*Yellow*	*Red*
Blue	*Yellow*	*Red*	*Green*	*Orange*	*Blue*
Green	*Orange*	*Blue*	*Yellow*	*Red*	*Green*
Yellow	*Red*	*Green*	*Orange*	*Blue*	*Yellow*
Orange	*Blue*	*Yellow*	*Red*	*Green*	*Orange*

Start.....one move....two moves..three moves..four moves...beginning

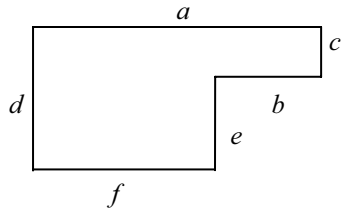
The cycle returns to the initial position every five moves. So, after twenty one moves the middle plate will be orange.

Some possibilities for further problems

P1 Instead of a thousand eggs, what is the probability that a pupil could eat these numbers of eggs in a minute? 1, 2, 5, 8, 20.

Q1 Will there be a question like this in the next PMC? What are the factors of 2013? Is 33 a factor? Is 671 a factor? When is the next prime year? (2017)

Q5 Pupils can write algebraic expressions for the lengths marked by d, *e* and *f* in terms of the other letters.
Can they write an algebraic expression for the area of the shape? In a few different ways?



Q9 Which of the shapes in the question have the largest number of faces, or vertices? Do you think a curved side can be called a face (e.g. on a cylinder, sphere)?

Q13 Here are some more nasty looking equations involving square roots!

a) $n = \sqrt{4} + \sqrt{4}$ $(n = 4)$ b) $5 = \sqrt{n} + 2$ $(n = 9)$
c) $8 = 13 - \sqrt{n}$ $(n = 25)$ d) $9 - \sqrt{n} = 5$ $(n = 16)$
e) $\sqrt{n} = n - 12$ $(n = 16)$

Q17 If today is a Wednesday, what will be the day of the week in 1000 days? In one million days (that is about 2747 years time) assuming the earth is still in one piece?

Q18 Suppose that the smoky bacon angle had the exact angles below. What would be the <u>minimum</u> number of people in the surveys? 20, 30, 40, 2.

Q20 How many tails would there be with the numbers of noses and legs shown below?

a) 15 noses and 50 legs → 10 tails b) 8 noses and 24 legs → 4 tails
c) 8 noses and 16 legs → 0 tails d) 8 noses and 32 legs → 8 tails
e) 10 noses and 8 legs → - 6 tails.

Using algebra for the last question (p is the number of people and d is the number of dogs) gives: $p + d = 10$; $2p + 4d = 8 \rightarrow p = 16$ and $d = -6$. The algebra allows us to enter a world with a negative number of dogs!

Q21 There are many different approaches to this question. Ask the pupils to compare and discuss their methods. Pupils could practise similar problems using the second method outlined in the above notes for the question.

Q24 If the question had asked how many Ls there were (instead of how many Vs) then the answer would have been two, with the third and fourth Ls not occurring until we reach 111,112,211,212... . Is this mathematics, or (merely) logical thinking?

Q25 Pupils could work out what the centre plate would be after 101 similar changes. They could look at the rules for using four plates, six plates (maybe examining the bottom plate in each of these as there will be no middle plate). Is there a general rule? If we had just three plates (red, blue, green) what would be the middle plate after 1000 moves? (red)
There are many other cyclic situations to examine; days of the week, hours on a clock, unit digits when counting, units digits in the times tables and traffic lights.

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